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Vector deformations of $\mathcal{N} = 4$ super Yang-Mills theory, pinned branes, and arched strings

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Abstract: We study a deformation of $\mathcal{N} = 4$ super-Yang-Mills theory by a dimension-5 vector operator. There is a simple nonlocal "dipole" field-theory that realizes this deformation. We present evidence that this theory is realized in the setting of "pinned-branes". The dipoles correspond to open strings that arch out of the brane. We find the gravitational dual of the theory at large N. We also discuss the generalization to the $(2, 0)$ theory.

Keywords: D-branes, AdS-CFT Correspondance.

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Contents

1. Introduction

Consider the superconformal $\mathcal{N}=4$ super-Yang-Mills on a noncommutative $\mathbb{R}^4$ (For recent developments in field-theory on noncommutative spaces see [1]–[3] and references therein). We will refer to it as $\mathcal{N}=4$ NCSYM. Let the noncommutativity be specified by a 2-form θ^{ij} such that the commutator of the coordinates on $\mathbb{R}^4$ is $[x^i,x^j]=i\theta^{ij}$. The parameter θ^{ij} has dimensions of Mass^{-2}. At low-energies, NCSYM can be described by augmenting the action with:

$$\int \theta^{ij}\mathcal{O}_{ij}(x)d^4x\,,$$

where $\mathcal{O}_{ij}$ is an operator of dimension 6 in the superconformal SYM on a commutative space. In the conventions such that the SYM lagrangian is:

$$\mathcal{L}_{SYM}=\mathrm{tr}\left\{\frac{1}{2g^2}\sum_{I=1}^{6}\partial_i\phi^I\partial^i\phi^I+\frac{1}{4g^2}F_{ij}F^{ij}+\frac{1}{2g^2}\sum_{I<J}[\phi^I,\phi^J]^2\right\}+\text{fermions}\,, \quad (1.1)$$

the bosonic part of the operator $\mathcal{O}_{ij}$ can be written as:

$$\mathrm{tr}\left\{\frac{1}{2g^2}F_{jk}F^{kl}F_{li}-\frac{1}{2g^2}F_{ij}F^{kl}F_{kl}+\frac{1}{g^2}F_{ik}\sum_{I=1}^{6}\partial_j\phi^I\partial^k\phi^I-\frac{1}{4g^2}F_{ij}\sum_{I=1}^{6}\partial_k\phi^I\partial^k\phi^I\right\}. \quad (1.2)$$

Here, g is the SYM coupling constant, F_{ij} is the $\mathrm{U}(N)$ field-strength, and ϕ^I ($I=1,\ldots,6$) are the scalars.

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The operator $\mathcal{O}_{ij}$ is neutral under the SU(4) R-symmetry and is part of a short representation of the supersymmetry algebra. It is the same operator that also describes the deformation of $\mathcal{N}=4$ SYM theory into the Born-Infeld theory, to first order in the NSNS 2-form B-field [4]. The multiplet of operators to which $\mathcal{O}_{ij}$ belongs also contains two vector operators in the representation **15** of SU(4), a self-dual tensor in the representation **10**, an anti-self-dual tensor in the representation $\overline{\mathbf{10}}$ as well as several fermionic operators (see for instance [5]–[7]). In this paper we will concentrate on one of the vector operators, $\mathcal{O}_i$ and $\widetilde{\mathcal{O}}_i$.

Since the deformation by $\mathcal{O}_{ij}$ can be extended to a complete theory with a simple description, i.e. NCSYM, one might expect that the deformation by $\mathcal{O}_i$ can be interpreted as the first term in a low-energy expansion of a simple theory too. This is indeed the case. The deformation by $L^i\mathcal{O}_i$ (where L^i is a constant vector) is the low-energy expansion of a nonlocal field-theory, the "dipole-theory," described in [8]. On $\mathbb{R}^{3,1}$, the non-perturbative description of NCSYM is more subtle — in particular, it is known that S-duality of non-commutative theories is problematic [9]–[11]. This has to do with the appearance of time-like noncommutativity [12]. The complete theory, NCOS, was described in [9, 11]. These issues are further discussed in [13]–[19]. It is likely that the dipole-theories need an extension as well, especially if L_i is time-like.

For simplicity, we can assume a Euclidean space. The bosonic part of the $\mathcal{N}=4$ SYM operator $\mathcal{O}_i$ can be calculated by changing to local variables (see [8] for more details). We can write it in $\mathcal{N}=1$ superfield notation as:

$$\mathcal{O}_i = \frac{i}{g_{YM}^2}\int d^2\theta\epsilon^{ab}\,\mathrm{tr}\left\{\sigma_i^{\alpha\dot{\alpha}}W_\alpha\Phi_a D_{\dot{\alpha}}\Phi_b + \Phi\Phi_a D_i\Phi_b + \text{c.c.}\right\}. \tag{1.3}$$

Here, the $\mathcal{N}=2$ vector-multiplet was decomposed into an $\mathcal{N}=1$ chiral field, Φ, and an $\mathcal{N}=1$ vector-multiplet whose field-strength is W_α. The hyper-multiplet was decomposed into two chiral multiplets Φ_a ($a=1,2$) and $\sigma_i^{\alpha\dot{\alpha}}$ are Pauli matrices). Note that $\mathcal{O}_i$ has conformal dimension 5.

On the other hand, nonlocal theories that are parameterized by a vector have been argued to appear on "pinned-branes" [20]. In this construction, the low-energy SYM that appears in the low-energy limit on branes is modified when they are put in an NSNS 3-form field-strength (see also [21] for related constructions). This effect can be realized by placing the branes near the center of a Taub-NUT space and turning on an NSNS 2-form flux at infinity. The NSNS flux has one direction along the Taub-NUT circle and another direction along the brane.

The purpose of the present work is to connect the pinned-brane theories to the dipole-theories. We will bring several pieces of evidence to indicate that they are one and the same theory:

- Both theories preserve the same amount of supersymmetry and they break the same part of the Lorentz group.

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- We will expand the IR part of the pinned-brane gravitational solution and show that it is a perturbation of $\mathbf{AdS}_5 \times \mathbf{S}^5$ by $\mathcal{O}_i$, in the context of the AdS/CFT correspondence [22]–[24]. This will also allow us to find the gravitational dual of the dipole-theories at large N analogous to the gravitational dual of the noncommutative gauge theories [25, 26].

- We will construct the dipoles as curved open strings that extend out of the brane and rotate in the directions transverse to the brane. They are stabilized by magnetic forces similar to those discussed in [27, 28].

- We will compactify on $\mathbb{T}^2$, turn on a magnetic field on the brane and show, from a BPS analysis, that the transverse fluctuations become massive. This agrees with the behavior of dipoles in a magnetic field and the conjecture that the fields that describe transverse fluctuations are dipoles.

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The paper is organized as follows. Section 2 is a review of the dipole-theories. In section 3 we review the pinned-brane constructions and expand the IR region as a perturbation of $\mathbf{AdS}_5 \times \mathbf{S}^5$. In this section we also present our conjecture for the gravitational dual of the dipole-theories. In section 4 we construct the dipoles as open strings that arch out of the brane and are held by (generalized) magnetic forces. In section 5 we compactify on $\mathbb{T}^2$ and study the behavior of the dipoles in a magnetic flux. In section 6 we present the generalization to the $(2,0)$ theory. We propose that there is a deformation of the $(2,0)$-theory to a theory that contains **discpoles** – membrane-like objects that generalize the dipoles.

2. Dipole Theories

Dipole-theories can be thought of as a generalization of field theories on commutative or noncommutative spaces. They are constructed by modifying the ordinary (or noncommutative) product of functions to the $\tilde{\star}$-product defined as follows. To each field, $\Phi(x)$, we assign a dipole-vector, L^i. The complex conjugate field, $\Phi^\dagger(x)$ is assigned the dipole-vector $-L^i$. If $\Phi_1(x)$ and $\Phi_2(x)$ have dipole-vectors L_1 and L_2 respectively, we define their dipole-product to be [8]:

$$(\Phi_1 \tilde{\star} \Phi_2)_{(x)} \equiv \Phi_1\left(x - \frac{L_2}{2}\right)\Phi_2\left(x + \frac{L_1}{2}\right). \tag{2.1}$$

For associativity, we have to make sure that the dipole-vector is additive, i.e. that $\Phi_1 \tilde{\star} \Phi_2$ is defined to have dipole-vector $L_1 + L_2$. Intuitively, the dipole field $\Phi(x)$ represents a dipole of length L starting at the point $x - L/2$ and ending at $x + L/2$. To see this, let us add a U(1) gauge field, $A(x)$, and define it to have dipole-vector zero. The covariant derivative is then:

$$\begin{aligned} D_i\Phi(x) &= \partial_i\Phi(x) - iA_i(x)\tilde{\star}\Phi(x) + i\Phi(x)\tilde{\star}A_i(x) \\ &= \partial_i\Phi(x) - iA_i\left(x - \frac{L}{2}\right)\Phi(x) + i\Phi(x)A_i\left(x + \frac{L}{2}\right). \end{aligned}$$

So $\Phi(x)$ transforms nontrivially under the subgroup $\mathrm{U}(1)_{(x-L/2)} \times \mathrm{U}(1)_{(x+L/2)}$ of the gauge group, where $\mathrm{U}(1)_{(x)}$ is the local transformation group at x.

To ensure associativity we can start with an ordinary theory that has a global U(1) symmetry and assign to the field Φ_I (I is an arbitrary index that labels the field) a dipole-vector in the form $L_I = Q_I L$ where L is a fixed vector, common to all the fields, and Q_I is the U(1) charge of Φ_I. More generally, working on $\mathbb{R}^d$, we can have global charges, Q_{Ia}, $a = 1, \ldots, l$ and a fixed $d \times l$ matrix, Θ^{ia} ($i = 1, \ldots, d$), such that the field Φ_I is assigned a dipole-vector $\sum_a \Theta^{ia} Q_{Ia}$.

The claim that field-theories on a noncommutative space are a special case of the dipole-theories can be interpreted in two ways. First, the $\tilde{\star}$-product can be defined to modify a noncommutative $\star$-product. We just have to interpret the products on the RHS of (2.1) as $\star$-products. Moreover, starting with a commutative space, we can take the charges Q_{Ia} above to be the components of the momentum of the field Φ_I and then $a = 1, \ldots, d$. If Θ^{ia} is chosen to be anti-symmetric we recover the familiar field-theory on a noncommutative $\mathbb{R}^d$. Let us also note that gauge theories on a noncommutative space can be recast in terms of bi-local fields [29] which might be reminiscent of the dipole-fields.[1]

In this paper we will concentrate on a dipole deformation of $\mathcal{N} = 4$ super-Yang-Mills (SYM) in 4D. The dipole-vectors will be correlated with a single U(1) charge, i.e. they are of the form $L_I = Q_I L$. The U(1) global symmetry is chosen as a subgroup of the R-symmetry, SU(4), that breaks it to SU(2) $\times$ U(1) and preserves $\mathcal{N} = 2$ supersymmetry. If we decompose the $\mathcal{N} = 4$ gauge field multiplet under $\mathcal{N} = 2$ supersymmetry, we get a vector-multiplet, V, and a hyper-multiplet, H. We take all the fields in the vector-multiplet to have dipole-vector zero and all the fields in the hypermultiplet to have dipole-vector L (their complex conjugate fields will have dipole vector $-L$). The lagrangian is obtained from the $\mathcal{N} = 4$ lagrangian by modifying the product to the $\tilde{\star}$-product, in a way that is similar to the construction of $\mathcal{N} = 4$ SYM on a noncommutative $\mathbb{R}^4$. We can take the gauge group to be either U(n) or SU(n). This is unlike SYM on a noncommutative $\mathbb{R}^4$, where the SU(n) theory is not well defined because two SU(n) gauge transformations can close to a U(n) gauge transformation when the product is changed to the $\star$-product [3]. In our case, the gauge-fields have dipole-vector zero and the gauge group is unmodified.

In the limit $L \to 0$, this particular dipole theory can be recast as a small deformation of ordinary $\mathcal{N} = 4$ SYM. The deformation operator is of the form $\int L^i \mathcal{O}_i(x) d^4x$ where $\mathcal{O}_i$ is the dimension five operator described in the introduction.

[1] We are grateful to N. Seiberg for pointing this reference out.

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The dipole-theories can be realized as brane configurations using a construction similar to that of [30]. Take an NS5-brane in directions $0, \ldots, 5$ and compactify the 6^{th} direction on a circle of radius R but such that the identification is: $(x_1, x_6) \sim (x_1 + L, x_6 + 2\pi R)$. Now take a D4-brane in directions $0, \ldots, 3, 6$. We are considering the limit $M_s^{-1} \ll R \ll L$ (where M_s is the string-scale). In this construction it is reasonable to expect that the open strings that connect the D4-branes on two sides of the NS5-brane will become dipoles.

In order to find the gravitational dual of the theory and especially in order to study the more interesting extension to the $(2,0)$ theory it will be useful to study another realization of the dipole theories where the branes on which the dipoles "live" are D3-branes. For that purpose we T-dualize along the 6^{th} direction to turn the D4-brane into a D3-brane and the NS5-brane into a Taub-NUT space.

3. Pinned branes

In [20], a class of non-Lorentz invariant field-theories was constructed by placing Dp-branes, M2-branes or M5-branes in a Taub-NUT space with NSNS or 3-form flux turned on at infinity. Specifically, consider type-II string-theory and take the four dimensional Taub-NUT solution:

$$ds^2 = R^2 U(dy - A_i dx^i)^2 + U^{-1}(d\vec{x})^2 , \qquad i = 1, \ldots, 3 , \qquad 0 \le y \le 2\pi , \qquad (3.1)$$

where

$$U = \left(1 + \frac{R}{|\vec{x}|}\right)^{-1} ,$$

and A_i is the gauge field of a monopole centered at the origin. The Taub-NUT solution is a fibration of a circle over $\mathbb{R}^3$ such that at $|x| \to \infty$ the circle has a constant radius, R. We now let n Dp-branes probe this geometry. The Dp-branes are points in the Taub-NUT space and extend along $(p+1)$ of the other 6 directions. We assume $p \le 4$. At infinity, we set the boundary condition on the NSNS 2-form B-field to approach a nonzero 2-form constant, b_{ij}. The 2-form is taken to have one direction along the Taub-NUT circle, and the other direction can be taken either parallel or transverse to the Dp-brane.

This configuration preserves 8 supersymmetries (i.e. $\mathcal{N} = 1$ for D4-branes, $\mathcal{N} = 2$ for D3-branes, $\mathcal{N} = 4$ for D2-branes and so on).

Let us first consider the case in which the B-field is set transverse to the Dp-brane. To be specific, let the D3 brane be oriented along $x^0, x^1, \ldots, x^3$. We keep it fixed at the origin of a Taub-NUT space whose nontrivial metric is along $x^6, x^7, \ldots, x^9$. The Taub-NUT circle is x^6. Using the results of [20] we can show that the B field as seen by the D3 brane is

$$B = h \ \tan \ \theta \ dx^5 \wedge (dx^6 + A_i dx^i) ,$$

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where $i = 7, 8, 9$ and $B(r \to \infty) = \tan\ \theta \equiv b$ is the value of B field at $r = \sqrt{x^i x^i} = \infty$. We also define

$$h^{-1} = \sin^2\theta + \left(1 + \frac{R}{r}\right)\cos^2\theta\,.$$

The string coupling is $g = e^{\phi} = \sqrt{h\,(1 + R/r)}$. (We have set the asymptotic value of the string coupling to unity.) Defining G_{ij} as the metric for the system, we can use a similar idea as in [20] to show that the D3 brane is "pinned". The pinning potential for this case will be

$$\frac{\sqrt{\det G}}{g} = \cos\ \theta = \frac{1}{\sqrt{1 + b^2}}\,.$$

The low-energy description of the D3-branes is U(n) SYM with a massive adjoint hyper-multiplet. The mass is given by:

$$m^2 = \frac{b^2}{1 + b^2}\,.$$

On the other hand, if the B-field with one leg is set parallel to the D3-branes, we get a nonlocal $(p+1)$-dimensional theory that is a deformation of SYM. To study the IR limit of the theory for a D3-brane we have to determine the low energy supergravity solution of the background. Let us take the direction of the B-field along the D3-brane to be the 1^{st}. Solving the equations of background supergravity we can show that the component of B field parallel to the D3 brane is given by

$$B_{16} = h\ \tan\ \theta\,,$$

where h is the same function as before. The behavior of the dilaton or the string coupling is again identical to the previous case. However the pinning potential is now

$$\frac{\sqrt{\det G}}{g} = 1$$

therefore there is no pinning! This is a generic phenomenon for branes in the background of Taub-NUT and B-fields with one leg parallel to the branes. The other leg of the B-field should be along the compact x^6 circle of Taub-NUT. The existence of Taub-NUT therefore reduces the worldvolume supersymmetry to $\mathcal{N} = 2$ and also ensures that we cannot gauge away the B-field.

In the large n limit we have a large number of D3 branes near a Taub-NUT singularity. However the point $r \to 0$ is a coordinate singularity and in the right choice of coordinate system $r = u^2$ Taub-NUT is actually a smooth manifold. Therefore the world volume theory of the D3 brane is a deformation of the $\mathcal{N} = 4$ supersymmetry which preserves $\mathcal{N} = 2$ supersymmetry. This deformation is due to the vector B_{1i} and it creates a scale in the theory. This scale is the dipole length and therefore from the supergravity point of view we have a deformed $\mathbf{AdS}_5$. In the rest of this section we will elaborate on this issue.

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For our configuration, in the absence of B field, the metric has two components $ds^2_{||} = ds^2_{0123}$ and $ds^2_{\perp} = ds^2_{45} + ds^2_{\mathrm{Taub-NUT}}$. The AdS background is then given by

$$ds^2 = H^{-1/2} ds^2_{||} + H^{1/2} ds^2_{\perp} \,,$$

where H is the harmonic function of the D3 branes. When we switch on a B field such that its asymptotic value is small the background metric gets deformed, for small r, to:

$$ds^2_{||} \to ds^2_{||} - rb^2 (dx^1)^2 \,, \qquad ds^2_{\perp} \to ds^2_{\perp} + O(r^2)(dx^6 + A_i dx^i)^2 \,.$$

Therefore near $r = 0$, the metric is $\mathbf{AdS}_5 \times \mathbf{S}^5$ and the NSNS B-field is $B_{16} = b\, r$.

Thus, this space is a deformation of the $\mathbf{AdS}_5 \times \mathbf{S}^5$ solution and the deformation approaches zero as $r \to 0$. In the AdS/CFT correspondence [22, 23, 24], such a deformation corresponds to a deformation of $\mathcal{N} = 4$ SYM by an irrelevant operator. Recall that a field that behaves as r^δ corresponds to a deformation $\int \mathcal{O}(x) d^4x$, where the operator $\mathcal{O}$ has dimension $4 + \delta$. In our case $\delta = 1$ and this operator is the same as the one in equation (1.3). In fact, in [5, 6, 7] the list of operators in $\mathcal{N} = 4$ SYM that belong to short SUSY representations was calculated from the AdS/CFT correspondence. It was found that there are two operators that are vectors and are descendants of the chiral primary $\mathrm{tr}\, \Phi^{(I_1} \Phi^{I_2} \Phi^{I_3)}$ where Φ^I are the scalars of $\mathcal{N} = 4$ SYM ($I = 1, \ldots, 6$) and $(I_1 I_2 I_3)$ denotes complete symmetrization. These vector operators are in the representation **15** of the R-symmetry group SU(4). They correspond to (1.3) and its magnetic dual. Thus, we have supporting evidence to the claim that the (un)pinned-branes in this case realize the dipole-theories.

4. The dipoles as arched strings

Consider the background of a Taub-NUT space before we have placed the D3-branes. In section (3) we have found the geometry of the Taub-NUT space with the B-field turned on at infinity. After the appropriate change of variables, $r = u^2$, it is easy to see that the origin, $r = 0$, is nonsingular and the 3-form NSNS flux $H = dB$ has a finite magnitude.

To complete the identification of the theory on the D3-brane in the above background with the dipole theory, we need to interpret the quanta of the dipole fields that arise on the brane. Clearly, the only objects that are charged under the U(1) of the D3-brane are open strings. However, open strings would seem to shrink to zero size by their own tension.

We propose the following resolution to this puzzle. Instead of considering a quantum of the dipole-field that has one unit of R-symmetry charge, let us consider a classical object with a large amount of R-symmetry charge. It is sufficient to

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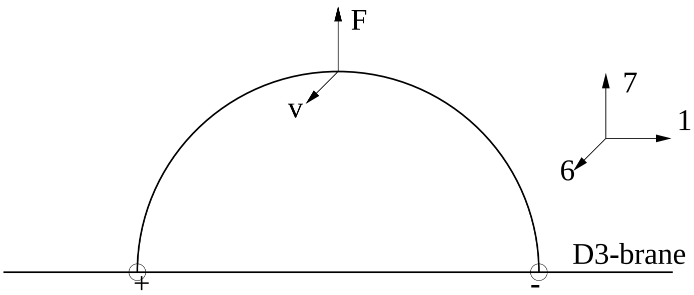

Figure 1: The dipole on a D3-brane is a string that arches out into the $6-7$ directions. The D3-brane is stretched along the 1^{st} direction (and directions $2, 3$ that are not shown) and is at the origin of the $6-7$ plane. The generalized magnetic force F is perpendicular to the velocity and the string.

restrict to the vicinity of the origin of the Taub-NUT space, so let us take a D3-brane in directions $0, 1, 2, 3$ and an NSNS 3-form H-flux in directions $1, 6, 7$. We will now construct an object with a large amount of angular momentum in the $6, 7$ plane that behaves like a dipole. In principle, R-symmetry corresponds to a simultaneous rotation in the $6-7$ and in the $8-9$ plane by the same angle. Thus, the object is formed by an open string that is parameterized as:

$$x_0 = \tau\,, \quad x_1 = \frac{L}{\pi}\sigma\,, \quad x_2 = x_3 = x_4 = x_5 = 0\,, \quad x_6 + ix_7 = x_8 + ix_9 = f(\sigma)e^{i\omega\tau}\,.$$

Here τ is the time and $0 \leq \sigma \leq \pi$ is the world-sheet parameter. $f(\sigma)$ is a profile that satisfies $f(0) = f(\pi) = 0$.

This configuration describes an open string that arches out of the $0, 1, 2, 3$ hyperplane of the D3-brane and into the $6, 7, 8, 9$ dimensions, with a profile $f(\sigma)$. It is rotating in the $6, 7, 8, 9$ dimensions with angular frequency ω. It is stabilized by magnetic forces similarly to those studied in [27, 28] for a D-brane moving in an RR-field strength. The magnetic force on a piece of string of length Δl moving in an H-field with velocity v is $Hv\Delta l$ and is perpendicular to the string and to v.

The string will be stable if the tension cancels the magnetic force. Thus the profile is determined by the equation of balance of forces:

$$\frac{\alpha' \frac{\pi^2}{L^2} f''}{1 + \frac{\pi^2}{L^2} f'^2} = H\omega f \sqrt{1 + \frac{\pi^2}{L^2} f'^2}\,. \tag{4.1}$$

The angular momentum is then determined to be:

$$J = \omega^2 \int_0^\pi f \sqrt{1 + \frac{\pi^2}{L^2} f'^2} d\sigma\,.$$

These equations are written in the non-relativistic approximation, $\omega f \ll 1$, in which case we can also neglect the centrifugal force. The full relativistic equations can be

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derived from the world-sheet action of the string in the background H-field, but we will not do that here. We also assume that the string is not radiating gravitational or B-field energy away. This can be justified by taking the limit $b \to \infty$ and noting that in this case there is a large rescaling of the metric near the D-brane, relative to the metric away from the brane, at infinity, by a factor of $\sqrt{1+b^2}$ (see section (5)). Note that the non-relativistic approximation is also consistent with the absence of radiation.

Equation (4.1) is a 2^{nd} order differential equation for $f(\sigma)$ that should be solved subject to the boundary conditions $f(0) = f(\pi) = 0$. This boundary condition should, in principle determine ω as a function of L and we could then calculate the angular momentum, J as a function of L.

Let us check how the profile $f(\sigma)$ behaves near the ends. Looking for a solution of the form $f(\sigma) \sim \sigma^\delta$ we find $\delta = 1/3$ so the string starts perpendicular to the D3-brane, as is required for equilibrium.

Note that the above discussion assumes that the angular momentum is large (so that the classical equations of motion can be used). We expect such objects to be heavy and of the order of magnitude of the string scale. The dipoles of the dipole-theories are light and can presumably be obtained by quantizing the open strings in the background of the H field.

5. Behavior in a magnetic flux

If we compactify a U(1) dipole theory on $\mathbb{T}^2$ of area A and put n_m units of magnetic flux on the $\mathbb{T}^2$, the boundary conditions for the dipole-fields acquire extra phases. As a result, the lowest Kaluza-Klein state of the dipoles has a mass of $\frac{n_m|L|}{A}$, in the non-compact directions. For an SU(N) theory, we have to replace n_m by $\frac{n_m}{N}$. With no magnetic flux, the dipoles are massless.

We can test these statements by calculating the masses from BPS arguments in the pinned-branes setting. It is convenient to compactify on $\mathbb{T}^6$. Let the radii of $\mathbb{T}^6$ be $R_1, \dots, R_6$. The Taub-NUT (TN) space becomes a Kaluza-Klein soliton. Let:

$$M_{TN} \equiv \frac{1}{g_s^2} M_s^8 R_1 \cdots R_5 R_6^2\,, \quad b \equiv M_s^{-2} B_{16}\,, \quad M_{D3} \equiv \frac{N}{g_s} M_s^4 R_1 R_2 R_3\,, \quad M_{dp} \equiv k R_6^{-1}.$$

We will eventually take the limit $R_1, \dots, R_5 \to \infty$. The BPS mass of a configuration of N D3-branes with k units of momentum along R_6^{-1} (that at the center of the Taub-NUT space becomes k units of R-symmetry charge) is:

$$\begin{aligned} m &= \sqrt{(1+b^2)M_{TN}^2 + M_{D3}^2 + M_{dp}^2 + 2\sqrt{(1+b^2)M_{D3}^2 M_{TN}^2 + b^2 M_{TN}^2 M_{dp}^2}} \\ &\longrightarrow \sqrt{1+b^2} M_{TN} + M_{D3} + \frac{b^2 M_{dp}^2}{(1+b^2) M_{D3}} + \cdots . \end{aligned}$$

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The arrow denotes the limit $M_sR_1, \ldots, M_sR_5 \gg 1$. Now, to turn on n_m units of magnetic flux in direction $1,2$ we define:

$$M_{D1} \equiv \frac{n_m}{g_s} M_s^2 R_3 \,.$$

The BPS mass formula, in the above limit, is (see [20] for more details):

$$m \to \sqrt{1+b^2} M_{TN} + M_{D3} + \frac{4bM_{D1}M_{dp}}{(1+b^2)M_{D3}} + \cdots \,,$$

where $(\cdots)$ are sub-leading corrections. So, the mass of k dipoles becomes:

$$\frac{4bM_{D1}M_{dp}}{(1+b^2)M_{D3}} = \frac{4bkn_m}{N(1+b^2)M_s^2 R_1 R_2 R_6} \,.$$

The dipole-length L can be extracted from this formula by comparing with the expected result of $n_m|L|/A$. But before we do that we have to take into account the rescaling of the metric due to the B-field. This rescaling is similar to the rescaling discussed in [3] and can be found by calculating the BPS mass of Kaluza-Klein excitations in the 1^{st} direction. The result is (see [20] for details) $m = 1/\widetilde{R}_1$ where $\widetilde{R}_1 = \sqrt{1+b^2}R_1$. So the rescaling in the 1^{st} direction is by a factor of $\sqrt{1+b^2}$ and we can identify the dipole length, L, as:

$$L = \frac{4b}{\sqrt{1+b^2} M_s^2 R_6} \,. \tag{5.1}$$

Note that with the magnetic flux turned on, the D3-brane becomes pinned to the center of the Taub-NUT space, in general. This follows from the fact that the transverse fluctuations, described by the dipole fields, are massive. There could, however, be special cases for which the D3-branes are not pinned. This happens when L is a rational fraction of one of the radii of $\mathbb{T}^2$ and n_m is a multiple of the denominator of this fraction.

The behavior of the branes in a magnetic field that we discussed above is a characteristic sign of the dipole-theories and we take it as another evidence for the identification of the dipole-theories with the low-energy description of the pinned branes.

From (5.1) we see that even when $b \to \infty$ the dipole length L is smaller than the string scale M_s^{-1} unless we take $R_6 \to 0$. This means that the T-dual picture discussed at the end of section (2) is better suited for describing large dipole-lengths. However, as we shall see in the next section, the generalization to the $(2,0)$ is a different story!

6. Generalization to the $(2,0)$ theory

There exists an interesting generalization of dipole-theories to a deformation of the $(2,0)$ theory that depends on a tensor $L^{\mu\nu}$. It can be similarly realized in the pinned-brane setting by putting M5-branes (in directions $1,\ldots,5$) inside a Taub-NUT space

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(that is homogeneous in directions $1,\dots,6$ and the Taub-NUT circle is in the 7^{th} direction) and turning on a 3-form C-field that at infinity approaches a constant C_{127}. The analysis is similar to the one we have performed in this paper and suggests that the theory on the pinned branes is parameterized by the tensor L^{12} that is proportional to C_{127}.

It is also amusing to conjecture that the theory has disc-like or membrane-like excitations with the boundary of the membrane in the $1-2$ plane and the area of the membrane being proportional to L^{12}. Those membranes are the generalization of the dipoles, but unlike a dipole whose boundary is two disconnected points, the boundary of an open membrane is a loop that can have a dynamics of its own if the membrane is light. We will call those objects "**discpoles**". The discpole-theory might be a simplified version of the noncommutative $(2,0)$-theory [31]–[33] or OM-theory [34]. (See also [35] for related ideas.)

We can repeat the analysis of section (5) and compactify this deformed $(2,0)$-theory on $\mathbb{T}^3$ with the analog of n_m units of magnetic flux that is a 3-form flux of the $(2,0)$-theory along $\mathbb{T}^3$. Defining:

$$M_{TN} \equiv M_p^9 R_1 \cdots R_6 R_7^2\,, \quad M_{M5} \equiv N M_p^6 R_1 \cdots R_6\,, \quad C \equiv M_p^3 C_{127}\,, \quad M_{dp} \equiv k R_7^{-1}\,,$$

and define

$$M_{M2} \equiv n_m M_p^3 R_4 R_5\,,$$

we find that the mass of the discpole in the presence of flux is:

$$\frac{4kCn_m}{N(1+C^2)M_p^3 R_1 R_2 R_3 R_7}\,.$$

The rescaling is now by a factor of $\sqrt{1+C^2}$ in both the 1^{st} and 2^{nd} direction. Therefore, the **discpole-tensor** is given by:

$$L^{12} = \frac{4C}{M_p^3 R_7}.$$

Now we see that in the limit $C \gg R_7 \to \infty$ the scale of the discpole-theories can be kept below M_p.

Viewing these discpoles as a source term in eleven dimensional supergravity we can, in principle, determine the detailed dynamics of the boundary.

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